World's simplest electric train

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World's simplest electric train

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We analyze the physics of the "world's simplest electric train." The "train" consists of a AA battery with a strong magnet on each end that moves through a helical coil of copper wire. The motion of the train results from the interaction between the magnetic field created by the current in the wire and the magnetic field of the magnets. We calculate the force of this interaction and the terminal velocity of the train due to eddy currents and friction. Our calculations provide a good illustration of Faraday's and Lenz's laws, as well as of the concepts of the Lorentz force and eddy currents.
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I. INTRODUCTION

A recent video on youtube shows how to build the "world's simplest electric train,"[1] consisting of a cylindrically shaped battery straddled by two magnets, that moves through a coil of wire. According to youtube, this video has been viewed nearly ten million times, giving some indication as to how captivating this simple device is. In this article, we analyze the dynamics of this system, including calculations that illustrate Faraday's and Lenz's laws, as well as incorporating the Lorentz force and eddy currents. Over the years, a large number of publications have appeared that describe simple electric motors, beginning with the classical Faraday and Ampère homopolar motors,[2–4] and that provide applications of eddy currents,[5–12] including electromagnetic braking, levitation, induction furnaces, and the curious behavior of a magnet falling inside a conductive non-magnetic metal tube.[13–21] The electric train we study in this paper is related to both the classical homopolar motors and the eddy-currents experiments.

The materials required to assemble the train are a battery, two strong magnets, and a long piece of copper wire formed into a coil. Figure 1 shows the train we constructed using spherical magnets and a 41-m long piece of unglazed copper wire with a total resistance of 3 Ω. The wire is wound into a left-handed helix with 597 turns of radius $R = 1.1$ cm, resulting in a finished coil length of 170 cm. The average separation between turns is $s = 2.85$ mm, and the resistance for one turn is approximately 0.005 Ω. The power source is a AA alkaline battery with a nominal voltage of 1.5 V, onto the ends of which we attach two spherical NdFeB magnets of radius $a = 0.95$ cm and residual magnetism $B_r = 1.24$ T.[22] The main reason for using spherical magnets is because of the dipole field existing outside such a geometry. Moreover, the spherical shape allows for a smoother motion through the coiled wire. To improve the stability of the arrangement, we have inserted washers between the battery and the spherical magnets.

The magnets are nickel-coated, allowing for a closed circuit to be formed between the battery and the portion of the coil between the magnets. Once the magnets touch the coil, the assembly is immersed in the magnetic field of a finite solenoidal current. The magnetic field **B** produced by the solenoid interacts with the magnetic dipole moment **m** of the spherical magnets, giving rise to a force that moves the train. The arrangement of the battery and the magnets, and the direction of the field created by the current in the solenoid, is shown in Fig. 2. Note that the two magnetic dipoles point in opposite directions. We will show that if **m** and **B** point in the same direction, the force on the magnet is in the direction of increasing **B**, whereas if they point in opposite directions, the force is in the direction of decreasing **B**. As we will see, this directionality is important for explaining the motion of the train. (We note that only one magnet is necessary for the motion to take place, as long as the other end includes something to close the circuit.) Once the train begins moving, it will quickly acquire a terminal speed due to eddy currents appearing in the coil. The force due to these currents is in the opposite direction of the train's motion and is proportional to its speed.

The force on the magnets due to the helical current will be calculated by two equivalent methods. In Sec. II A, we calculate the force as the gradient of the potential energy of a permanent magnetic dipole in a magnetic field **B**. In Sec. II B, the same result is obtained by calculating the Lorentz force on the solenoid due to the magnetic field of the magnets, and then invoking Newton's third law to get the force on the train. In Sec. III, the force on the train due to eddy currents is calculated, which then allows us to determine the terminal velocity of the train.

The analysis presented here is within the capabilities of students in a first course on electromagnetism. We have

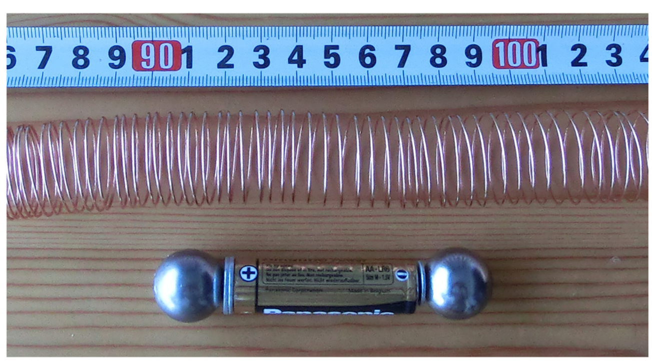

Fig. 1. Photograph of the "train" arrangement and of the coiled copper wire (the "track").

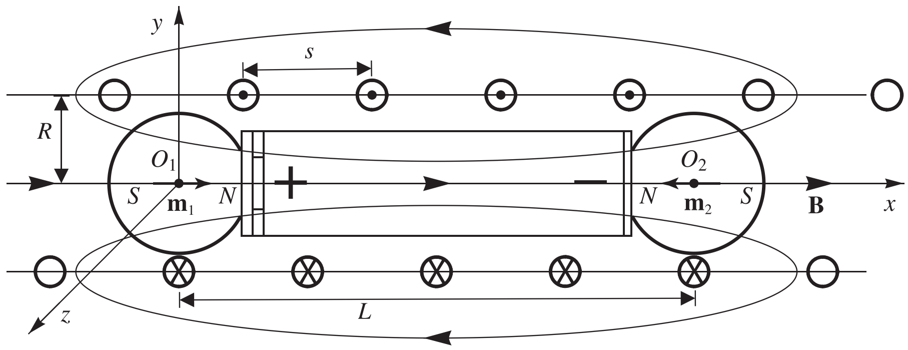

Fig. 2. Schematic diagram of the train. The direction of the electric current on a portion of the coil due to the battery, the magnetic dipoles, as well as the magnetic field B produced by the current are shown. The north and south poles of each magnetic dipole are denoted by *N* and *S*, respectively.

included a sample problem in the Appendix related to the calculations shown in this paper.

II. THE FORCE ON THE TRAIN DUE TO A HELICAL CURRENT

The force on the train due to the current in the helix will be calculated using two different methods. The first involves taking the gradient of the potential energy function.

A. Gradient of dipole potential energy

For a spherical magnet of radius a and (uniform) magnetization $\mathbf{M}$, the magnetic field inside is given by $\mathbf{B} = (2/3)\mu_0\mathbf{M}$, with a magnitude equal to the residual magnetism[22] $B_r = 1.24$ T. Meanwhile, the field outside is that of a pure dipole[23] of magnetic moment $\mathbf{m} = (4/3)\pi a^3\mathbf{M}$, with a magnitude, in this case, of $m = 5.31$ A m^2.

The potential energy of a permanent magnetic dipole $\mathbf{m}$ in a magnetic field $\mathbf{B}$ is given by[23]

$$U = -\mathbf{m}\cdot\mathbf{B}. \tag{1}$$

In our case, we have two spherical magnetic dipoles, d_1 and d_2, with their centers located at O_1 and O_2, respectively (see Fig. 2). We consider a right-handed reference frame with the origin at O_1, the x-axis in the direction O_1O_2, and the y-axis opposite the direction of gravity. The distance between O_1 and O_2 is $L = 7$ cm, and in this interval, there is an average of $N = 24.58$ turns of wire.

In the specified coordinate system, the dipole moment of the magnet at O_1 is $\mathbf{m}_1 = (m, 0, 0)$ and the dipole moment at O_2 is $\mathbf{m}_2 = (-m, 0, 0)$. The force on a magnetic dipole is $\mathbf{F} = -\nabla U$, where U is the potential energy given in Eq. (1). If $\mathbf{B} = (B_1, B_2, B_3)$ is the magnetic field created by the current I along the helix, then for dipole d_1 we have $U = -mB_1(x)$ while for dipole d_2 we have $U = mB_1(x)$.

Now, the magnetic field $\mathbf{B}$ at a point $\mathbf{x} = (x, 0, 0)$ on the axis of our helix h is given by the Biot–Savart law[24] as

$$\mathbf{B} = \frac{\mu_0 I}{4\pi}\int_h \frac{d\mathbf{x}'\times(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}, \tag{2}$$

where $\mu_0 = 4\pi\times 10^{-7}$ N A^{-1} is the magnetic permeability of free space, and the integration is over the helix specified by

$$\mathbf{x}' = (x', -R\cos(kx'), -R\sin(kx')), \tag{3}$$

with $0 \le x' \le L$ and $k = 2\pi/s = 2\pi N/L$, $s = L/N$ being the average separation between turns (see Fig. 2). A straightforward calculation of the first component of the above integral gives

$$B_1(x) = \frac{\mu_0 I}{4\pi}kR^2\int_0^L \frac{dx'}{\left[(x-x')^2+R^2\right]^{3/2}} = \frac{\mu_0 IN}{2L}\left(\frac{x}{\sqrt{x^2+R^2}} - \frac{x-L}{\sqrt{(x-L)^2+R^2}}\right). \tag{4}$$

Observe that this expression for $B_1(x)$ is completely general, valid for any finite separation between turns—we do not assume the usual approximation that the turns are closely spaced (ideal solenoid).[24]

By taking the derivative of Eq. (4), we obtain

$$\left.\frac{dB_1(x)}{dx}\right|_{x=0} = -\left.\frac{dB_1(x)}{dx}\right|_{x=L} = \frac{\mu_0 IN}{2L}\left[\frac{1}{R} - \frac{R^2}{(L^2+R^2)^{3/2}}\right]. \tag{5}$$

The force on dipole d_1 is then given by

$$F(0) = -\left.\frac{dU(x)}{dx}\right|_{x=0} = m\left.\frac{dB_1(x)}{dx}\right|_{x=0}, \tag{6}$$

whereas for dipole d_2 it is

$$F(L) = -\left.\frac{dU(x)}{dx}\right|_{x=L} = -m\left.\frac{dB_1(x)}{dx}\right|_{x=L}. \tag{7}$$

Using Eq. (5), we see that $F(0) = F(L)$, so the total force F is given by

$$F = CI, \tag{8}$$

where

$$C = \frac{\mu_0 mN}{L}\left[\frac{1}{R} - \frac{R^2}{(L^2+R^2)^{3/2}}\right]. \tag{9}$$

B. The Lorentz force

We can also calculate the force on the train due to the current in the helix using the Lorentz force. The total magnetic force $\mathbf{F}$ on the helix h due to the field $\mathbf{B}$ created by the two magnets is given by the Lorentz force as[23]

$$\mathbf{F} = I\int_h d\mathbf{x}' \times \mathbf{B}. \tag{10}$$

In our case, due to the symmetry of the configuration and the direction of the current (see Fig. 3), the total force is in the negative x-direction. In cylindrical coordinates (ρ, θ, x), we have $d\mathbf{x}' = d\rho\,\hat{\boldsymbol{\rho}} + \rho\,d\theta\,\hat{\theta} + dx\,\hat{\mathbf{x}}$ and $\mathbf{B} = B_\rho\,\hat{\boldsymbol{\rho}} + B_\theta\,\hat{\theta} + B_x\,\hat{\mathbf{x}}$, where $\hat{\boldsymbol{\rho}}$, $\hat{\theta}$, and $\hat{\mathbf{x}}$ are unit vectors associated with these coordinates.

To get the force in the x-direction, we need to take the x-component of the cross-product $d\mathbf{x}' \times \mathbf{B}$, so

$$F = I\int_h (B_\theta\,d\rho - B_\rho\,\rho\,d\theta) = -IR\frac{2\pi N}{L}\int_0^L B_\rho\,dx, \tag{11}$$

where we have used the fact that $\rho = R$ and $x = (L/2\pi N)\theta$ on the helix, which gives $d\rho = 0$ and $d\theta = (2\pi N/L)dx$.

To calculate $B_\rho(x)$, we first calculate the radial component $B_{\rho 1}(x)$ due to dipole d_1 at a point of horizontal coordinate x of the helix. The magnetic field of a dipole of moment $\mathbf{m}$ at a point $\mathbf{b}$ is given by[23]

$$\mathbf{B} = \frac{\mu_0}{4\pi b^3}\left[3(\mathbf{m}\cdot\hat{\mathbf{b}})\hat{\mathbf{b}} - \mathbf{m}\right], \tag{12}$$

where $b = |\mathbf{b}|$ and $\hat{\mathbf{b}} = \mathbf{b}/b$. Thus, the radial component $B_{\rho 1}$ due to dipole d_1 is given by $B_{\rho 1} = \mathbf{B}\cdot\hat{\boldsymbol{\rho}}$. For d_1 we then get

$$B_{\rho 1}(x) = \frac{3\mu_0 mRx}{4\pi(R^2 + x^2)^{5/2}}, \tag{13}$$

while for d_2 a similar calculation gives

$$B_{\rho 2}(x) = -\frac{3\mu_0 mR(x - L)}{4\pi\left[R^2 + (x - L)^2\right]^{5/2}}. \tag{14}$$

Substituting $B_\rho = B_{\rho 1} + B_{\rho 2}$ into Eq. (11), we then find

$$\begin{aligned} F &= -\frac{\mu_0 INmR^2}{2L}\left\{-\frac{1}{(R^2 + x^2)^{3/2}} + \frac{1}{\left[R^2 + (x - L)^2\right]^{3/2}}\right\}_0^L \\ &= -\frac{\mu_0 INm}{L}\left[\frac{1}{R} - \frac{R^2}{(L^2 + R^2)^{3/2}}\right] = -CI, \end{aligned} \tag{15}$$

with C given by Eq. (9).

Thus, by Newton's third law, the force on the magnet is along the positive x-axis and is the same as calculated in Eq. (8), as expected.

III. FORCE DUE TO EDDY CURRENTS

Due to the motion of the magnets through the coiled wire, eddy currents will appear in the coil. These currents produce a magnetic force on the train that opposes its motion. This force increases with velocity, and therefore, the velocity is a self-limiting physical quantity. From this fact, we deduce that the train will reach a terminal velocity v.

Eddy currents are formed only when the conductor allows closed loops of current to flow. In our case, eddy currents will appear only in the section of the coil between the magnets because the battery acts as a low resistance conductor. Therefore, the eddy current I_E will be calculated as the ratio of the net electromotive force (emf) $\mathcal{E}$ over this section of the coil to the total resistance R_T of the closed circuit. We can calculate the emf $\mathcal{E}$ by summing the corresponding emf contributions $\mathcal{E}_1$ and $\mathcal{E}_2$ due to the dipoles d_1 and d_2, respectively.

We begin by applying Lenz's law to dipole d_1: $\mathcal{E}_1 = -d\phi_1/dt$, where ϕ_1 is the magnetic flux of the field created by dipole d_1. The surface whose boundary is the closed loop is a helicoid H (see Fig. 4), which has the parametric equation $\mathbf{r}(x,\rho) = (x, -\rho\cos(kx), -\rho\sin(kx))$, for $0 \le x \le L$, $0 \le \rho \le R$, and $k = 2\pi/s = 2\pi N/L$. The magnetic flux through the helicoid of the field $\mathbf{B}$ produced by dipole d_1 is given by $\phi_1 = \int_H \mathbf{B}\cdot d\mathbf{A}$, with

$$d\mathbf{A} = \left(\frac{\partial\mathbf{r}}{\partial x} \times \frac{\partial\mathbf{r}}{\partial\rho}\right)dx\,d\rho = (k\rho, -\sin kx, \cos kx)\,dx\,d\rho. \tag{16}$$

Assuming the train is at the location shown in Fig. 2 at time $t = 0$ and moving with a constant speed v in the x-direction, the field $\mathbf{B}$ of dipole d_1 at a point (x, y, z) of the helicoid will depend on time, and, using Eq. (12), is given by

$$\mathbf{B}(\mathbf{r}(t)) = \frac{\mu_0}{4\pi\left[x^2(t) + \rho^2\right]^{3/2}}\left[\frac{3mx(t)}{x^2(t) + \rho^2}\mathbf{r}(t) - \mathbf{m}\right], \tag{17}$$

where $\mathbf{r}(t) = (x(t), y, z)$ is the position vector of a point of the helicoid with respect to the position of d_1 at time t. Note that

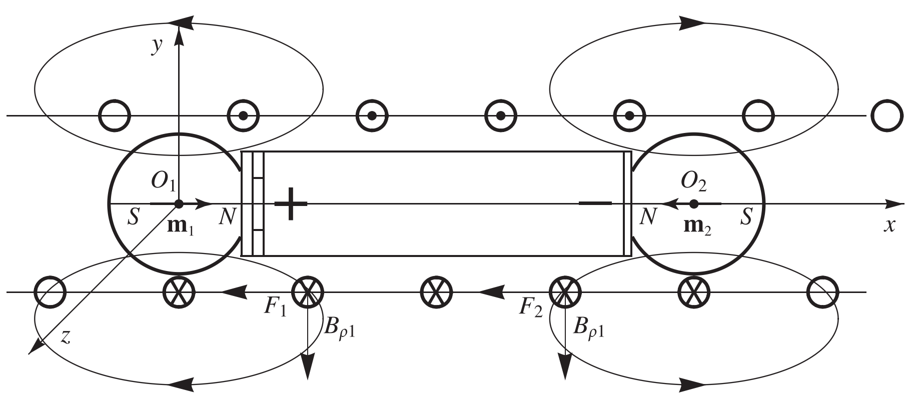

Fig. 3. Schematic of the train with some lines of the magnetic fields created by the magnetic dipoles. The radial components $B_{\rho 1}$ and $B_{\rho 2}$ of these fields on two turns, as well as the Lorentz forces F_1 and F_2 on a differential element of these turns due to $B_{\rho 1}$ and $B_{\rho 2}$ are also represented.

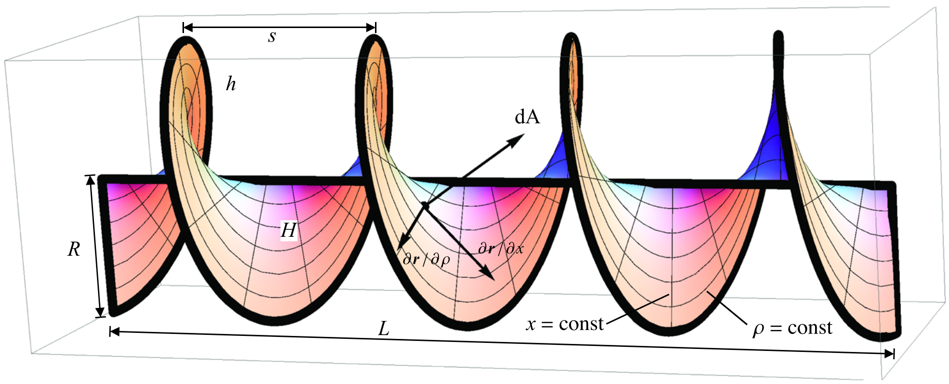

Fig. 4. Illustration of the closed loop (thick black line) formed by the coiled helix h, the magnets, and the battery. H is the helicoid through which we calculate the magnetic flux. The curves $\rho = \text{constant}$ and $x = \text{constant}$, their tangent vectors, and the vector $d\mathbf{A}$ are also represented.

$\mathbf{r}(0) = (x, y, z)$ represents an arbitrary, *time-independent* point on the helix at time $t = 0$, and given that the train moves with speed v in the positive x-direction, we have $x(t) = x - vt$ so that $dx(t)/dt = -v$. (This is simply a statement that points on the helix move with speed v in the negative x-direction with respect to the train.)

The emf $\mathcal{E}_1$ thus generated by dipole d_1 will be

$$\mathcal{E}_1 = -\frac{d\phi_1(t)}{dt} = -\int_H \frac{d\mathbf{B}(\mathbf{r}(t))}{dt} \cdot d\mathbf{A}. \tag{18}$$

Using the chain rule, we have

$$\frac{d\mathbf{B}(\mathbf{r}(t))}{dt} = -v\left(\frac{dB_1}{dx}, 0, 0\right), \tag{19}$$

which leads to

$$\begin{aligned}\mathcal{E}_1 &= v\int_0^L \int_0^R \frac{dB_1}{dx} k\rho \, d\rho \, dx \\ &= v\frac{\mu_0 mN}{2L}\int_0^L \int_0^R \frac{d}{dx}\left[\frac{\rho\left(2x^2 - \rho^2\right)}{\left(x^2 + \rho^2\right)^{5/2}}\right] d\rho \, dx \\ &= v\frac{\mu_0 mN}{2L}\int_0^L \frac{d}{dx}\left[\frac{R^2}{\left(x^2 + R^2\right)^{3/2}}\right] dx \\ &= -\frac{1}{2} vC,\end{aligned} \tag{20}$$

where C is given by Eq. (9).

A similar argument for dipole d_2 leads to an identical expression for the emf $\mathcal{E}_2$, as expected by symmetry. Therefore, the eddy current is given by

$$I_E = \frac{\mathcal{E}}{R_T} = \frac{2\mathcal{E}_1}{R_T} = -\frac{C}{R_T} v. \tag{21}$$

As can be seen in the above equation, the eddy current opposes the current produced by the battery. By Eq. (8), the total electromagnetic force on the train is given by $F = C(I + I_E)$. The terminal velocity is found when the total force is zero, that is, when

$$C\left(I - \frac{C}{R_T} v\right) - Mg\mu_K = 0, \tag{22}$$

where Mg is the weight of the train and μ_K is the coefficient of kinetic friction. Therefore, we obtain a linear relationship between the terminal velocity v, the open-circuit voltage of the battery V, and the effective total resistance (which includes the internal resistance of the battery) of the closed circuit R_T

$$v = \left(\frac{1}{C}\right) V - \left(\frac{Mg\mu_K}{C^2}\right) R_T. \tag{23}$$

The above equation can be used to calculate the velocity v from the values $R = 1.1$ cm, $L = 7$ cm, $N = 24.58$, $M = 77.5$ g, $V = 1.51$ V, $m = 5.31$ A m^2, $\mu_K = 0.45$, and the value of R_T. To estimate the coefficient of kinetic friction μ_K, we used a discharged battery in the train and measured its acceleration through the helix when pulled by a hanging weight. Unfortunately, it is difficult to estimate the value of R_T due to the instability and jumps of the contacts between the magnets and the coil. We have estimated its value by summing the battery's internal resistance of $r_{\text{int}} = 0.31\ \Omega$ (see Ref. 25), the resistance of 24.58 turns of the copper coil (0.123 Ω), and the estimated contact resistance of the magnets. To estimate this last resistance, we have measured the voltage at the battery terminals while current is flowing through the coil and the train is held still, obtaining 0.94 V. Using Ohm's law, we get $R_T - r_{\text{int}} = 0.511\ \Omega$, so the approximate value for the contact resistance is 0.388 Ω. Thus we obtain $R_T = 0.821\ \Omega$ and, using Eq. (23), calculate a terminal velocity of $v = 86$ cm/s. This calculated terminal velocity is a little higher than our measured value of $v = 69.5$ cm/s. We believe the difference is due to the contact resistance being slightly higher when the train is moving due to the changing points of contact. Agreement with the measured terminal velocity would require a contact resistance of 0.413 Ω (giving $R_T = 0.881\ \Omega$).[26]

Given the difficulty in establishing some of the parameters in our model, we consider the agreement between the measured and observed terminal velocity to be quite good.

IV. SUMMARY

The motion of the "world's simplest electric train" has been studied. The force on the train is due to the interaction between the magnetic field created by the current in the coiled wire and the magnetic field of the magnets. We have calculated this force in two different ways. Eddy currents due to the motion of the magnets have also been calculated, and we have shown how such currents result in the train having a terminal velocity. The value calculated for this terminal velocity agrees well with the observed value.

The materials used in this experiment are readily available, and the theory is accessible to students of electromagnetism at the undergraduate level, providing a useful example of the electromotive force on a magnetic dipole subject to a divergent magnetic field.

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APPENDIX: SUGGESTED PROBLEM

We provide here a sample problem (and solution) related to the calculations in this paper that is appropriate for students.

Problem: Calculate the force on the train due to eddy currents by using the fact that the power dissipated by this force is equal to the power dissipated by the eddy currents.

Solution: The power dissipated by the force F_E due to eddy currents is $P = vF_E$, and is equal to the power dissipated by the ohmic losses of the eddy currents; that is,

$$P = vF_E = -I_E^2 R_T, \tag{A1}$$

where R_T is the total resistance of the closed circuit, and $I_E = -Cv/R_T$ is the eddy current calculated in Sec. III. We thus obtain

$$F_E = -\frac{I_E^2 R_T}{v} = -\frac{C^2 v}{R_T}, \tag{A2}$$

which coincides with $F_E = CI_E$ as obtained from Eq. (8).

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[1]The “Worlds simplest electric train” video can be found on youtube at <https://www.youtube.com/watch?v=J9b0J29OzAU>.

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[25]The internal battery resistance r_{int} has been calculated using Ohm’s law by $r_{int} = (V/V_L - 1)R_L$, where V and V_L are the values of the voltage across the battery terminals for an open circuit voltage, and for a known load resistance R_L, respectively.

[26]Although in this paper we have assumed that the center of the spherical magnets are on the axis of the coiled wire (what we have denoted the x-axis), we note that this is only an approximation. In fact, because the radius of the magnets ($a = 9.5$ mm) is smaller than radius of the helix ($R = 11$ mm), the centers of the magnets are $R - a = 1.5$ mm below the x-axis. We have investigated how this fact affects the terminal velocity. To accomplish this, we have repeated all the calculations taking this fact into account, and integrating numerically. The calculation of the field due to the current of the battery in any point $\mathbf{x} = (x, y, z)$ of the interior of the coiled wire is given by using Eq. (2). We have numerically calculated the force on dipole d_1 by using $\mathbf{F}_1 = -\nabla(-\mathbf{m}\cdot\mathbf{B})_{\mathbf{x}=\mathbf{x}_0}$ for $x_0 = (0, -R + a, 0)$, performing the gradient before numerical integration. For dipole d_2, we get $\mathbf{F}_2 = \mathbf{F}_1$, and for $V = 1.51$ V and $R_T = 0.846\ \Omega$, we have found the total force on the two dipoles $\mathbf{F} = 2\mathbf{F}_1 = (0.383, 0.00, -0.037)$ in Newtons. Note that in addition to the driving force in the x-direction, the force has a small lateral component, which is due to the left-right asymmetry of the coiled wire. The terminal velocity v calculated with this corrected force now increases by 11.1%. We have also numerically calculated the effect of the correction on eddy currents. When both corrections are taken into account, the value of v increases by 8.8%. We conclude that the asymmetry of the magnets location results in an error of less than 10% when using Eq. (23).